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A nonlinear time series analysis using two-stage genetic algorithms for streamflow forecasting

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Abstract:

Streamflow forecasting is very important for the management of water resources: high accuracy in flow prediction can lead to more effective use of water resources. Hydrological data can be classified as non-steady and nonlinear, thus this study applied nonlinear time series models to model the changing characteristics of streamflows. Two-stage genetic algorithms were used to construct nonlinear time series models of 10-day streamflows of the Wu-Shi River in Taiwan. Analysis verified that nonlinear time series are superior to traditional linear time series. It is hoped that these results will be useful for further applications. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS nonlinear time series; genetic algorithm; streamflow forecast; threshold autoregressive model; bilinear time series; autoregressive condition heteroscedasticity model

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INTRODUCTION

Water is one of mankinds necessities and global environmental changes have forced the realization that water resources are not inexhaustible. Thus, many countries are now focusing on enhancing the efficiency of water utilization. If the accuracy of flow prediction can be improved, reservoir operation can also be improved and a more effective utilization of water resources achieved.

Hydrological information for time series analysis is essential for water resources management. Streamflow simulations and forecasts are also important for water resources planning and allocation. As most hydrological time series analyses involve nonstationary, nonGaussian and nonliner data, these data need to be transformed before they can be used in many stochastic models for hydrological time series. But even after such transformations, the hydrological information is still nonlinear and it is necessary to alter the linear assumption. However, several nonlinear models developed in past decades may provide a solution for the use of hydrological time series models. Therefore, this study investigatess nonlinear time series analyses and predicts their suitability for 10-day streamflows of the Wu-Shi River in Taiwan.

There has been a growing interest in studying nonlinear time series models. Tong and Lim (1980) developed the threshold autoregressive model (TAR), which adopts different linear regressive models. TAR has been used in several studies (Tsay, 1989; Tong, 1990; Potter, 1995; Chen and Lee, 1995; Chen *et al*., 1997; Wong and Li, 1998; Koop and Potter, 1999; Dijk *et al*., 1999; Cai *et al*., 2000; McDowall, 2002; Kapetanios, 2003; Liu and Enders, 2003; Simon *et al*., 2004; Chan *et al*., 2004). In addition, modified threshold models were developed, such as the threshold moving-average model (De Gooijer, 1998) and the functional-coefficient autoregressive model (Chen and Tsay, 1993). In 1981, Subba developed a moving autoregressive model with random parameters and called it a bilinear time series model. However, a great number of fluctuations in data can be detected when using this model. Further applications were developed by Weiss (1986), Wu and Shih (1992), Wu and Hung (1999), and Chen and Wu (2001). Engle (1982) first proposed the ARCH (autoregressive conditional heteroscedasticity) model. He suggested that the variance of a certain time series can be modelled directly in terms of past observations, and that the variance of the process is time variant. ARCH models have been widely used in finance, economics, and ecology, fields in which the phenomenon of nonlinearity is reflected in variance or higher-order moments. The modified ARCH model was further developed into models such as the EGARCH model by Nelson (1991). These models were applied in many studies, including those conducted by Bera and Higgins (1997), Chun and Li (2001), and Li and Lin (2004). Haggan and Ozaki (1981) and Ozaki (1982) proposed a novel model called the exponential autoregressive model. This model was employed by Arango and González (2001) and Taylor and Sarno (2002). In 1982, Nicholls and Quinn proposed a random coefficient AR model, which was further applied by Tseng *et al*. (1995), Yu and Chiao (2000), Yu (2002), and Yu and Tseng (2004).

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Recently, numerous researchers have applied nonlinear time series models to hydrosystems because most hydrological measurement data are highly nonlinear. For example, Jian *et al*. (1998) utilized the nonlinear time series model TAR to analyse monthly groundwater flow. Fleming *et al*. (2002) adopted the Fourier transform for hydrological data estimation.

Holland (1975) first proposed genetic algorithms (GA), which proved useful in solving different types of combinatorial optimization algorithms and were applied to hydrosystems in many studies, including Chen (1997), Wang (1997), Rauch and Harremoes (1999), Chang and Chang (2001), Chang *et al*. (2003), Kuo and Liu (2003), and Smakhtin and Batchelor (2005).

This present study tries to combine nonlinear time series with Gas, using the GA as a tool for solving nonlinear time series. It is very difficult to simultaneously compute model formulas and their parameters in nonlinear time series. Therefore, a nonlinear time series was used in this study to better simulate and manipulate the varying characteristics of hydraulic systems, whereby two-stage GAs were adopted to linearize the problems associated with nonstationary, nonGaussian, and nonlinear data. Several models were examined based on the kind of data they can interpret, including the autoregressive moving-average (ARMA), the outlier detection, the autoregressive condition heteroscedasticity (ARCH), the threshold autoregressive (TAR), the threshold autoregressive moving-average (TARMA), and the bilinear (BL) models. Scientific Computing Association (SCA) software was used for linear time series simulation, and Econometric Views (E-Views) for the ARCH model. A GA adopting two-stage operation was applied for the parameters in nonlinear time series models. For the nonlinear hydrological data, this study adopted piecewise linear TAR and TARMA models, which consider different sets as change points. The characteristics of the multiplicative AR and MA in the bilinear time series model causes very large amplitude variations. This model was, therefore, used to simulate the wide variation of streamflows during the wet season. In particular, the hydrological data showed obvious variations, prompting the use of the ARCH model to further analyse residuals. Finally, 10-day streamflow data gathered from the Da-Do Bridge station on the Wu-Shi River were used in the improved models, which integrate hydrological properties, a novel nonlinear, and several traditional linear time series models.

The major objective of this study is to investigate the nonlinear time series TAR and TARMA models. TAR and TARMA models are piecewise linear models and the threshold value characteristics could divide into different linear sets. This study utilizes the features of the two model to separate high and low streamflows, and then explains the phenomenon in the case of Taiwan, which has very obvious dry and wet seasons.

The multiplied alternately in AR and MA items were the BL model items. Like the Equation (5) bp and bq. If the items' parameters is more large, the large-amplitude phenomenon will be more obvious. This paper investigates use of the BL model to simulate the high streamflows in Taiwan caused by typhoons and torrential rain giving high streamflows.

The streamflow of wet season in Taiwan have the different quantities of variance. The study want to combine the ARCH, TAR and TARMA model characteristic try to explain the real phenomenon.

THEORY

Linear time series models

ARMA model. The ARMA model may be applied to general seasonal and nonseasonal cases. The process is illustrated in the following: let y_t be a time series following a general ARMA(p, q) process,

$$y_t = C + (\phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p}) + (a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}) \quad (1)$$

where C is a constant that determines the mean of the process, a_t is an i.i.d. normal error with a zero mean and a standard deviation of σ_a, p is the autoregressive order of the process, q is the moving average order of the process, ϕ_j is the autoregressive coefficient, and θ_j denotes the moving average coefficient.

Outlier detection model. Time series observations are sometimes influenced by interruptive events, such as typhoons, droughts, or data errors. As a consequence, spurious observations occur that are inconsistent with the rest of the series. Such observations are usually referred to as outliers.

Nonlinear time series models

Threshold autoregressive (TAR) model. Tong (1990) provides an excellent review of the properties of the TAR model. One of his research objects is a piecewise linear model in the space of y_{t-d}. A time series y_t is said to follow a TAR model with threshold variable y_{t-d} if it satisfies

$$y_t = \phi_0^{(k)} + \sum_{i=1}^{p_k} \phi_i^{(k)} y_{t-d} + a_t^{(k)} \quad \text{for} \quad r_{k-1} \le y_{t-d} < r_k, \quad (2)$$

where $k = 1, \cdots, g$ and d are positive integers. $a_t^{(k)}$ is a sequence of independent and identical distribution (i.i.d.), normal random variates with mean zero, and variance σ_k^2. The real number r_j satisfies $-\infty = r_0 < r_1 < \cdots < r_g = \infty$ and forms a partition of the space y_{t-d}. The positive integer d is commonly referred to as the delay (or threshold lag) of the model. The partition denotes the TAR model in (2) by TAR($g; p_1, \cdots, p_g$). Consider a TAR($2; p_1, p_2$) process:

$$y_t = \begin{cases} \phi_0^{(1)} + \sum_{i=1}^{p_1} \phi_i^{(1)} y_{t-i} + a_t^{(1)} & \text{for } y_{t-d} \le r \\ \phi_0^{(2)} + \sum_{i=1}^{p_2} \phi_i^{(2)} y_{t-i} + a_t^{(2)} & \phantom{\text{for }} y_{t-d} > r \end{cases} \quad (3)$$

Threshold autoregressive moving average (TARMA) model. To combine the TAR model with the ARMA model, we consider a TARMA process:

$$y_t = \begin{cases} \phi_0^{(1)} + \sum_{i=1}^{p_1} \phi_i^{(1)} y_{t-i} + a_t^{(1)} + \sum_{j=1}^{q_1} \theta_j^{(1)} a_{t-j}^{(1)} \\ \phi_0^{(2)} + \sum_{i=1}^{p_2} \phi_i^{(2)} y_{t-i} + a_t^{(2)} + \sum_{j=1}^{q_2} \theta_j^{(2)} a_{t-j}^{(2)} \end{cases} \quad (4)$$

Bilinear (BL) model. The bilinear model is very effective in describing a wide class of signals. The bilinear model of the BL(p, q, bp, bq) time series is defined as follows:

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-p\log(i)} + a_t - \sum_{j=1}^{q} \theta_j a_{t-qlqg(j)} + \sum_{k=1}^{bp} \sum_{l=1}^{bq} \beta_{kl} y_{t-bplag(k)} a_{t-bqlag(l)} \quad (5)$$

where $\beta_{kl}, k = 1, \cdots, bp, l = 1, \cdots, bq$, are parameters and a_t is an iid $N(0, \sigma_a^2)$ innovation that drives the bilinear process.

Autoregressive conditional heteroscedasticity (ARCH) model. Assume $Y_t = \sigma_t \varepsilon_t$ for $-\infty < t < \infty$, where the random variables ε_t are independent and identically distributed with zero mean and unit variance. ε_t is independent of $\{Y_{t-i}, i \geq 1\}$,

$$\sigma_t^2 = c + \sum_{i=1}^{p} a_i Y_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2, \quad (6)$$

where $c > 0, a_i \geq 0, b_j \geq 0, p \geq 0$ and $q \geq 0$, the latter two quantities of course being integers. If $q \geq 1$ the model is a generalized ARCH, or GARCH.

This study combines TAR, TARMA, and ARCH models,

$$Y_t = a_0 + \gamma_0 D_t + \sum_{i=1}^{p} (a_i Y_{t-i} + \gamma_i D_t Y_{t-i}) + \varepsilon_t, \quad (7)$$

$$\varepsilon_t | I(t-1) \sim N(0, h_t),$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \gamma_1 D_t,$$

where $\alpha_0 > 0, 0 < \alpha_1 < 1$, and

$$D_t \begin{cases} 0 & if \quad Y_t < r \\ 1 & if \quad Y_t \geq r \end{cases}.$$

Genetic algorithms

Genetic algorithms (GAs) are search procedures that use random choices as a tool to guide a highly exploitative search through the coding of a parameter space. They combine the survival of the fittest among string structures, yet randomize information exchange to form a search algorithm with some of the innovative flair of human search (Goldberg, 1989).

GAs serve to mimic biological phenomena that occur in natural selection. To evaluate the suitability of a derived solution, an objective function is required. The objective function is chosen in such a way that well-fitted strings are accorded high fitness values. The evolution starts from a set of corded solutions (chromosomes) and proceeds from generation to generation through genetic operations: reproduction, crossover and mutation.

FORECASTING MODELS

Data analysis

Typhoons and plum rains pass through Taiwan, bringing torrential rainfall in the wet season between May and October; the dry season lasts from November to April. There are great differences in streamflows between the two seasons. A 10-day period of hydraulic information is adopted for water resources management in Taiwan, thus, a 10-day streamflow time series was used in this study.

The hydrological station in the Wu-Shi watershed is shown in Figure 1. The 10-day streamflow data from 1988 to 2001 gathered at the Da-Do Bridge Station, Wu-Shi watershed, was used for primary analysis in the forecasting models. These data were further grouped into an appraisal set, including 360 sets of streamflow data from 1989 to 1998, and a calibration set, including 108 sets of data from 1999 to 2001. Figure 2 and Figure 3 demonstrate the 10-day flow hydrographs for both sets. Calibration and verification of the 10-day streamflow mean are shown by the horizontal line.

The linear time series model

This study adopted the ARMA and outlier detection models to build a linear time series analysis model. The resulting model (Figure 4) served as a control for comparison with the nonlinear models.

Nonlinear time series models

Nonlinear time series models TAR, TARMA, and BL were used in this study. However, in these nonlinear time series it is very difficult to simultaneously compute model formulas and their parameters. Therefore, the nonlinear time series models were simulated by integrating them with the search technology of Gas, adopting two-stage genetic algorithms to linearize this problem.

First, we used the Bayesian information criterion (BIC) standard and let all generations match the nonlinear time series formulas. BIC is the criterion for selection of a time series to match and diagnose the simulated results for parameter estimates and standard errors. Therefore, the characteristic of the BIC criterion can help to fit the best type of nonlinear time series. In the study, the BIC index was adopted to calculate the fitness function and to choose the best formulas using GAs.

According to the result of the first GA stage, the simulations were run to obtain the best parameters corresponding the nonlinear time series models. The

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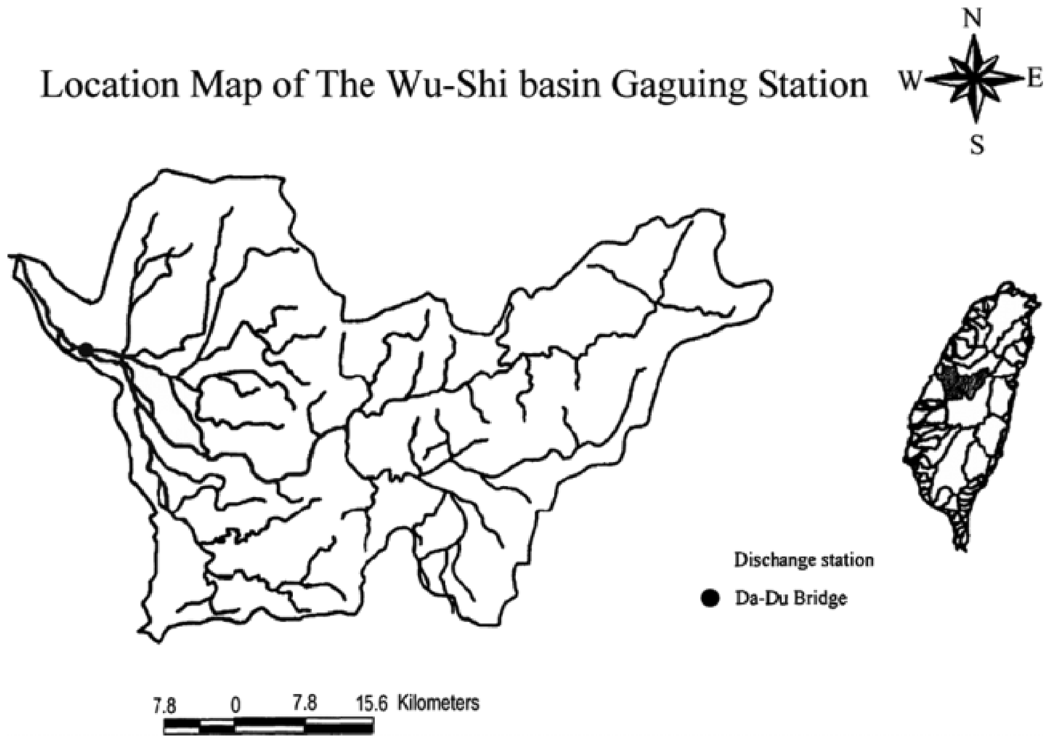

Figure 1. The hydrological station in the Wu-Shi watershed

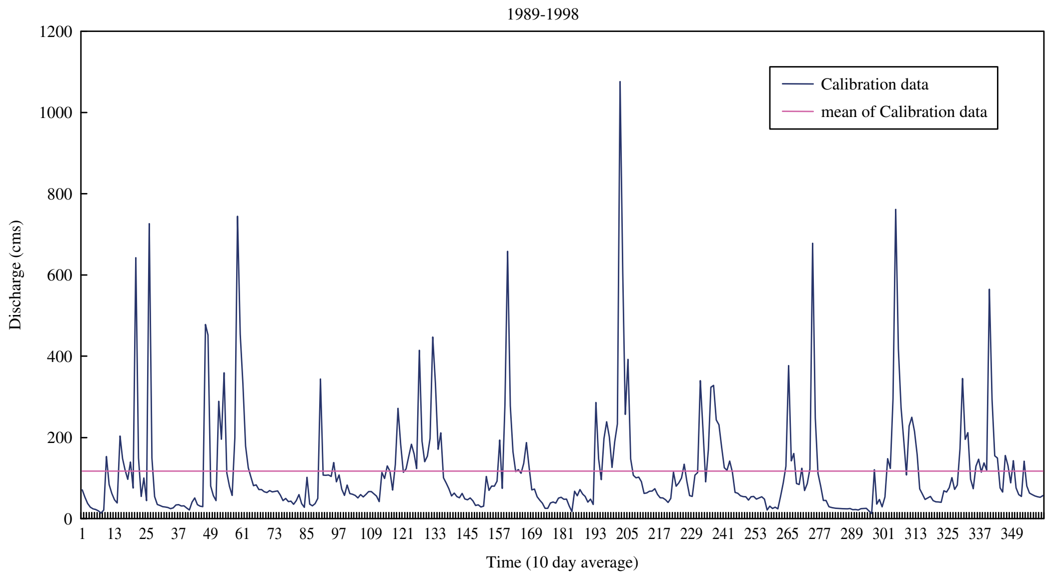

Figure 2. Results of calibration of 10-day streamflow data obtained at Da-Do Bridge between 1989 and 1998

coefficient of efficiency (CE) was selected as assessment indicator to determine the fitness function for the second stage genetic algorithms. CE is an assessment indicator that can estimate how well the simulation results match the real models.

The BIC (Bayesian Information Criterion) used to help choose the best time series models is defined as follows:

$$BIC(M) = n \ln \hat{\sigma}_\varepsilon^2 - (n - M)\ln(1 - \frac{M}{n}) + M \ln n + M \ln \left[\frac{(\hat{\sigma}_z^2/\hat{\sigma}_\varepsilon^2 - 1)}{M} \right]$$

where n is the number of data items, M is the number of parameters, $\hat{\sigma}_z^2$ is the variance of the sample, and $\hat{\sigma}_\varepsilon^2$ is the variance of the residual.

In GA terminology, an initial input, called a chromosome, is necessary. As in most optimization problems, a random generator is used to produce chromosomes for the initial optimization implementation. Fitness evaluation, reproduction, mutation, and selection were performed for each epoch. The algorithm was stopped when a specified criterion providing an estimate of convergence was reached. The framework of the GA for the nonlinear time series is shown in Figure 5.

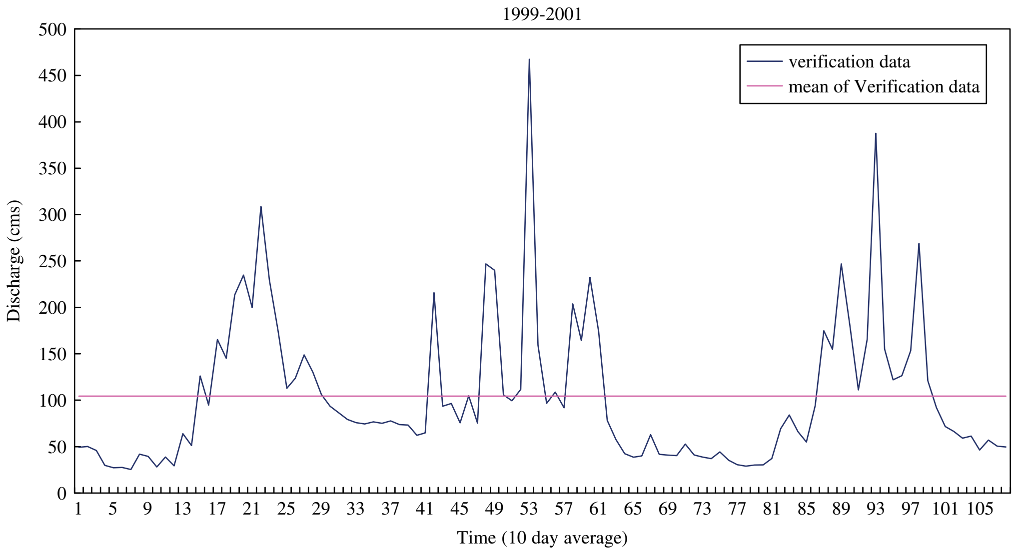

Figure 3. Results of verification of 10-day streamflow data obtained at Da-Do Bridge between 1999 and 2001

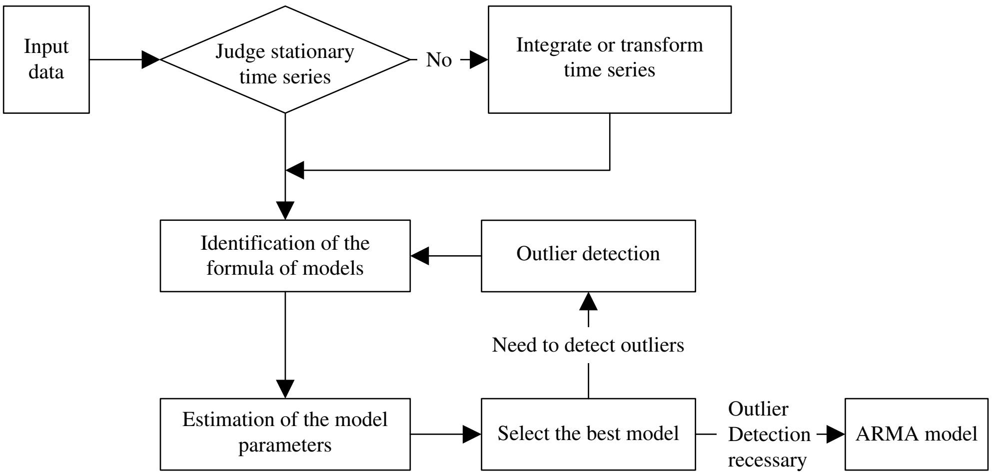

Figure 4. Flowchart of the linear time series models

The generational evolutionary algorithm was implemented identically and the parameters are described in Table I.

Average 10-day streamflows exhibit a higher variance in residual value during the wet season. Therefore, a TAR threshold was defined to separate high flow values, and an explanation variable was added to construct the ARCH time series model. This procedure is illustrated in Figure 6.

Table I. Parameters of the GAs

Fitness	Sum of the integers in chromosome
Model	Binary
Initial population	Random
Maximum length	50
Population size	500
Crossover probability	0·8
Mutation probability	0·001
First-stage fitness function	Minimum value of BIC (Bayesian Information Criterion)
Second-stage fitness function	Maximum value of CE (Coefficient of Efficiency)

Assessment Indicator

Seven identification indicators were used in this study. The assessment indicators 1–5 were implemented for model calibration and indicators 1–7 for model verification.

Indicator 1: Coefficient of efficiency (CE)

$$CE = 1 - \frac{\sum (Q_{obs} - Q_{est})^2}{\sum (Q_{obs} - \overline{Q}_{obs})^2} \tag{8}$$

where Q_{est} is the estimated flow from the forecast model (cms), Q_{obs} denotes the observed flow (cms), and $\overline{Q}_{obs}$ is the mean observed flow (cms).

Indicator 2: Error of peak discharge (EQ_p)

$$EQ_P = \frac{Q_{pest} - Q_{pobs}}{Q_{pobs}} \tag{9}$$

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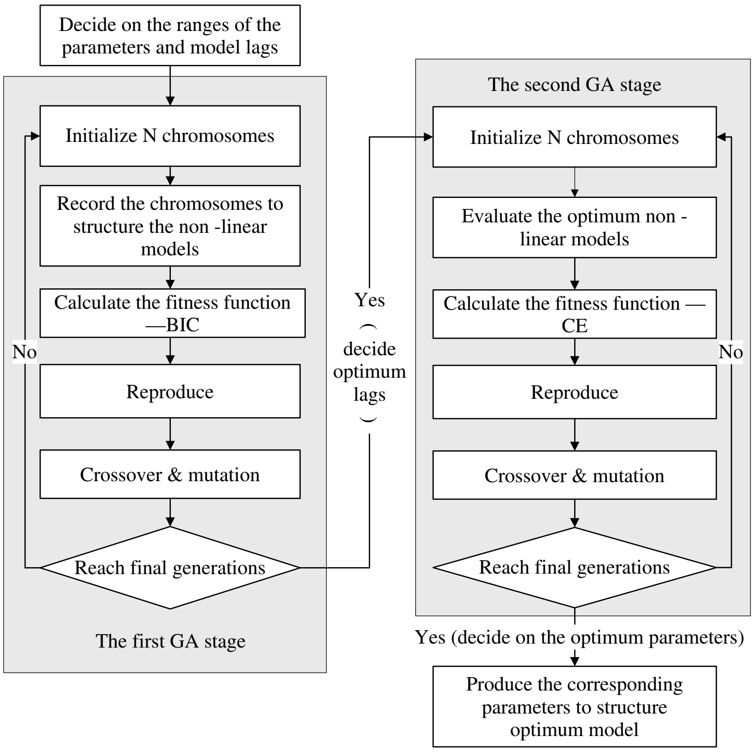

Figure 5. Flowchart of the non-linear time series models

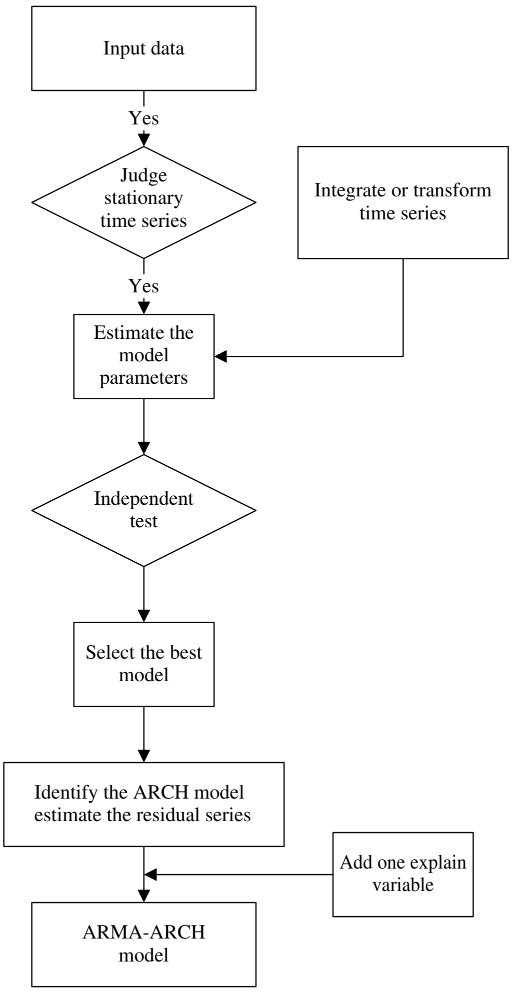

Figure 6. Flowchart of the autoregressive conditional heteroscedasticity model

where Q_{pobs} is the observed peak discharge (cms) and Q_{pest} denotes the forecast peak discharge from the model (cms).

Indicator 3: Error of time to peak ($\mathrm{ET_p}$)

$$ET_p = T_{pest} - T_{pobs} \tag{10}$$

where T_{pest} is the estimated arrival time of the peak discharge from the forecast model, and T_{pobs} denotes the arrival time of the observed peak discharge.

Indicator 4: Mean absolute error (MAE)

$$MAE = \frac{1}{M}\sum |Q_{obs} - Q_{est}| \tag{11}$$

where M is the number of estimated runoff points and MAE is the difference between estimated and observed flow.

Indicator 5: Mean absolute percentage error (MAPE)

$$MAPE = (\frac{1}{M}\sum \left|\frac{Q_{obs} - Q_{est}}{Q_{obs}}\right|)100\%, \tag{12}$$

MAPE is a measurement of the accuracy of the estimated and observed flow.

Indicator 6: Coefficient of persistence (PC)

$$PC(k) = 1 - \frac{\sum (Q_{obs} - Q_{est})^2}{\sum (Q_{obs} - Q_{obs}(k))^2} \tag{13}$$

where $PC(k)$ indicates the estimated coefficient of persistence at the kth period and the observation value at k periods before is as present observation $Q_{obs}(k)$.

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Indicator 7: Coefficient of extrapolation (EC)

$$EC(k) = 1 - \frac{\sum (Q_{obs} - Q_{est})^2}{\sum (Q_{obs} - Q_{est}(k))^2} \quad (14)$$

where $EC(k)$ is the estimated coefficient of extrapolation at the kth period and $Q_{est}(k)$ is the estimated value at the kth period extrapolated from the previous kth and $(k-1)$th observed values.

APPLICATION AND RESULTS

Linear times series analyses

The times series statistical package, SCA, was used for linear time series analysis. The results from the ARMA model and the outlier detection were obtained as follows:

(1) ARMA Model

The best model, ARMA (2(1, 18),1(5)), was obtained using the BIC values

$$Y_t = \underset{(6{\cdot}71)}{76{\cdot}76} + \underset{(9{\cdot}77)}{0{\cdot}47}Y_{t-1} - \underset{(-2{\cdot}30)}{0{\cdot}11}Y_{t-18} + a_t + \underset{(-3{\cdot}74)}{0{\cdot}20}a_{t-5} \quad (15)$$

In the above equation, the t-value in brackets are the results from the T-test, indicating a significant difference if the value is larger than 1.96. The t-value is the statistic by T-test.

(2) Outlier detection

After application of outlier detection the estimated parameters were in steady state and obtained using the following expression:

$$Y_t = \underset{(8{\cdot}53)}{76{\cdot}43} + \underset{(7{\cdot}33)}{0{\cdot}47}Y_{t-1} - \underset{(-1{\cdot}61)}{0{\cdot}11}Y_{t-18} + a_t + \underset{(-3{\cdot}30)}{0{\cdot}21}a_{t-5} \quad (16)$$

The linear model ARMA and outlier consider seasonal fluctuations. Equations (15) and (16) indicate the significance of Y_{t-18}, and the equations demonstrate the strong influence of a period of one-half year as 18 ten-day periods were chosen by the model as the optimum. The seasonal fluctuation of 18 ten-day periods fits the physical phenomenon, since the hydraulic streamflow data can be divided into that for wet and dry seasons.

Nonlinearity test

Integrating Ramsey's RESET auxiliary regression into Equation (15) and proposing h = 2, the following expression was obtained:

$$Y_t = 76{\cdot}76 + 0{\cdot}47Y_{t-1} - 0{\cdot}11Y_{t-18} + \delta_2\hat{Y}_t^2 + a_t + 0{\cdot}20a_{t-5} \quad (17)$$

where $\hat{Y}_t$ indicates the estimated value. The original null hypothesis $H_0 : \delta_2 = 0$, has to be rejected for P-values less than 0·05. In this case study, the F-test resulted in F = 4·93 and $P = 0{\cdot}027$. Hence, the null hypothesis H_0 was rejected, and existence of the H_0 nonlinear term δ_2 confirmed. The following expression was again regressed:

$$Y_t = \underset{(6{\cdot}10)}{62{\cdot}42} + \underset{(15{\cdot}53)}{0{\cdot}69}Y_{t-1} - \underset{(-1{\cdot}73)}{0{\cdot}07}Y_{t-18} \underset{(-2{\cdot}17)}{-0{\cdot}0004}\hat{Y}_t^2 + a_t + \underset{(5{\cdot}50)}{0{\cdot}29}a_{t-5} \quad (18)$$

From the coefficients of the nonlinear terms in Equation (18) it is obvious that nonlinearity characteristics do exist even when the value of the nonlinear term is not high. The parameters are insignificant when h > 2.

Equation (18) confirms that the hydraulic time series was nonlinear. We utilize the nonlinear time series models to simulate and forecast the streamflow.

Nonlinear time series analyses

There were two stages in the construction of the GA model; the number of initial population genes was 50. In the first stage, the BIC was applied to formulate the optimized equation (e.g. $Y_{t-1}, Y_{t-2}, Y_{t-5}\ldots$) for the TAR, TARMA, and BL models. In the second stage a CE value was utilized to fit the best parameters of the optimized equation formula from the first stage. In addition, the algorithm ranking method for reproduction and uniform cross for crossover were accepted: a crossover rate of 0·8 and a mutation rate of 0·01 were selected through trial and error. The highest number of generations when picking the chromosomes was set a 50. The ARCH model was constructed using the E-views package.

(1) TAR and TARMA model

As stated above, the optimization results from the GA were implemented in the TAR model as follows:

$$Y_t = \begin{cases} 40 + 0{\cdot}584Y_{t-1} + a_{1,t} & Y_{t-d} \le r \\ 287{\cdot}5 + 0{\cdot}5Y_{t-1} - 0{\cdot}079Y_{t-4} + a_{2,t} & Y_{t-d} > r \end{cases} \quad (19)$$

where d = 1, and $r = 320$.

The Equation of (19),(20),(22),and (24) were divided into two piecewise linear model by d and r. The positive integer d is commonly referred to as the delay (or threshold lag) of the model. The r is the threshold value of the model.

Owing to the larger variations of 10-day streamflows between dry and wet seasons, variables, such as MA terms, were integrated into the TAR model, and the maximum order was 5. The results of the TARMA model were as follows:

$$Y_t = \begin{cases} 96 + 0{\cdot}42Y_{t-1} + 0{\cdot}071Y_{t-2} \\ -0{\cdot}135Y_{t-4} + 0{\cdot}50a_{t-1} + 0{\cdot}211a_{t-2} \\ -0{\cdot}125a_{t-3} + a_{1,t} & Y_{t-d} \le r \\ 511{\cdot}5 - 0{\cdot}1563Y_{t-4} + 1{\cdot}87a_{t-2} \\ -0{\cdot}1504a_{t-5} + a_{2,t} & Y_{t-d} > r \end{cases} \quad (20)$$

where d = 1, and $r = 281$.

(2) BL model

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The optimization implemented through GA in the bilinear model can be determined as follows:

$$\begin{aligned} Y_t = {} & 84{\cdot}63 + 0{\cdot}20Y_{t-3} + 0{\cdot}34Y_{t-4} \\ & + a_t + 0{\cdot}25a_{t-1} + 0{\cdot}20a_{t-4} + 0{\cdot}11a_{t-5} \\ & + 0{\cdot}0005Y_{t-1}a_{t-1} + 0{\cdot}0008Y_{t-1}a_{t-5} \\ & + 0{\cdot}001Y_{t-2}a_{t-3} + 0{\cdot}0001Y_{t-3}a_{t-2} \\ & + 0{\cdot}0017Y_{t-4}a_{t-2} \end{aligned} \tag{21}$$

(3) ARCH model

High streamflows in the wet season show higher variations. Hence in this study the thresholds from the TAR and TARMA models were utilized to separate the high streamflow data. In addition, a virtual variable, D_t, was proposed as an explanatory variable. The primary function of D_t is to increase the significance of the value over the threshold in the model. The results from this model are shown in Equations (22)–(25). The values in brackets indicate the identifier of the T-test. In additions, the virtual variable shows significance if the modulus of D_t is greater than 1·96.

(i) If threshold $r = 320$

$$\begin{aligned} Y_t = {} & \underset{(14{\cdot}84)}{57{\cdot}65} + \underset{(20{\cdot}30)}{0{\cdot}25Y_{t-1}} - \underset{(-2{\cdot}17)}{0{\cdot}05Y_{t-18}} \\ & + \underset{(5{\cdot}40)}{D_t(153)} + \underset{(6{\cdot}86)}{D_t(0{\cdot}73)Y_{t-1}} + a_t + \underset{(4{\cdot}11)}{0{\cdot}11a_{t-5}} \\ & \varepsilon_t | I(t-1) \sim N(0, h_t) \\ h_t = {} & \underset{(14{\cdot}94)}{1088{\cdot}88} + \underset{(7{\cdot}50)}{0{\cdot}82a_{t-1}^2} + \underset{(2{\cdot}02)}{D_t(8804)} \end{aligned} \tag{22}$$

$$D_t \begin{cases} 0 & Y_t < r \\ 1 & Y_t \geq r \end{cases} \tag{23}$$

(ii) If threshold $r = 281$

$$\begin{aligned} Y_t = {} & \underset{(12{\cdot}67)}{59{\cdot}19} + \underset{(18{\cdot}18)}{0{\cdot}25Y_{t-1}} - \underset{(-2{\cdot}41)}{0{\cdot}06Y_{t-18}} \\ & + \underset{(15{\cdot}88)}{D_t(287)} + \underset{(2{\cdot}83)}{D_t(0{\cdot}18)Y_{t-1}} + a_t + \underset{(5{\cdot}15)}{0{\cdot}17a_{t-5}} \end{aligned} \tag{24}$$

$$\begin{aligned} & \varepsilon_t | I(t-1) \sim N(0, h_t) \\ h_t = {} & \underset{(13{\cdot}73)}{1261} + \underset{(6{\cdot}19)}{0{\cdot}64a_{t-1}^2} + \underset{(4{\cdot}94)}{D_t(5460)} \end{aligned}$$

$$D_t \begin{cases} 0 & Y_t < r \\ 1 & Y_t \geq r \end{cases} \tag{25}$$

From the above implementation it can be seen that different threshold values represent different explanation variables. Finally, $r = 281$ is preferable for calibration.

DISCUSSION

Calibration and verification

Calibration and verification of the linear and nonlinear time series are shown in Tables II, III and IV, and Figures 7–20:

(1) The characteristics of the dry and wet seasons cannot be captured using the ARMA model. Hence the results of this simulation were not very representative of the true situation. Consequently, this study utilized outlier detection technology to capture the characteristics and occurrence of outliers, and then re-adjusted the series. Finally, it was found that the number of outliers was much less than in the original data. Thus, there was no significant improvement in the calibration and verification of the results.
(2) Even though the modified linear models, which used outlier detection technology, performed much better than ARMA in identification, the performance of CE and the values of the forecast capacity, PC and EC, although better than those of the ARMA model—were still not very representative owing to the fact that the occurrence and size of outliers cannot be forecasted.
(3) Regardless of data identification and verification, the nonlinear TAR and TARMA models performed much better than the linear models.
(4) A stable forecast of the total water amount is more important than the forecast of peak values in water resources management. Accordingly, for CE, MAPE, and MAE values, the nonlinear models TARMA,

Table II. Calibration results of each model

Assessment Index	*CE*	*EQp*	*ETp*	*MAE*	*MAPE*
ARMA	0·2886	−0·4716	1	60·456	0·7440
Outlier	0·2886	−0·4716	1	60·45	0·7437
TAR	0·7311	−0·2657	1	42·54	0·5008
TARMA	0·7541	−0·4872	154	40·61	0·4841
ARCH-320	0·7625	−0·2411	0	41·35	0·4759
ARCH-281	0·7338	−0·3797	102	42·14	0·4785
BL	−0·2487	−0·0203	1	104·43	1·4099

Table III. Verification results of each model

Assessment Index	*CE*	*EQp*	*ETp*	*MAE*	*MAPE*	*PC*	*EC*
ARMA	0·3817	−0·3595	1	41·82	0·5349	0·2902	0·7279
OUTLIER	0·3022	−0·4007	1	37·99	0·3521	0·1989	0·6929
TAR	0·5884	0·1152	40	33·84	0·3891	0·5275	0·8189
TARMA	0·6058	0·0946	32	33·33	0·3878	0·5474	0·8265
ARCH-320	0·4784	0·1152	40	38·38	0·3830	0·4011	0·7704
ARCH-281	0·4973	0·1152	40	37·36	0·3782	0·4228	0·7788
BL	0·0938	−0·4191	1	61·28	0·9102	−0·0403	0·6012

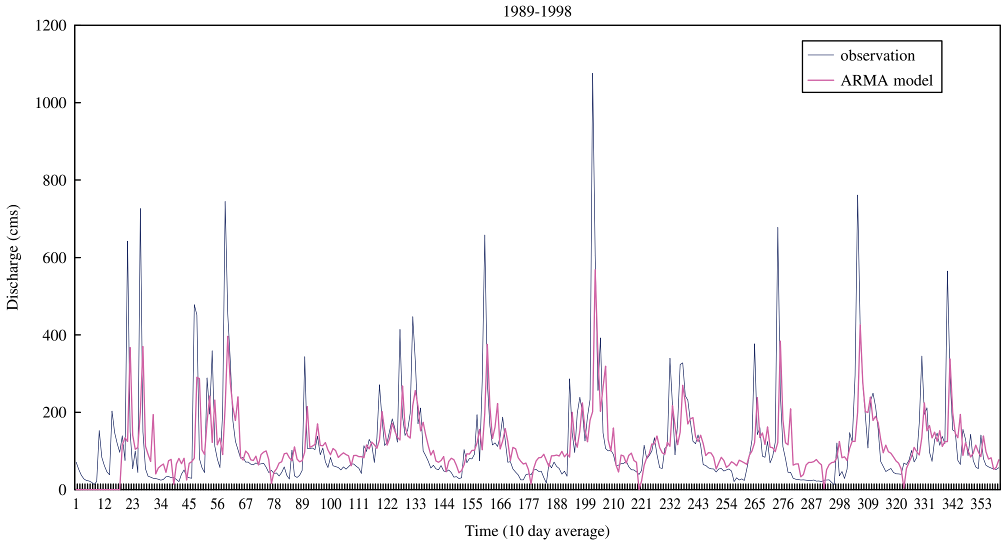

Figure 7. Calibrated results for 10-day streamflows between 1989 and 1998 using the ARMA model

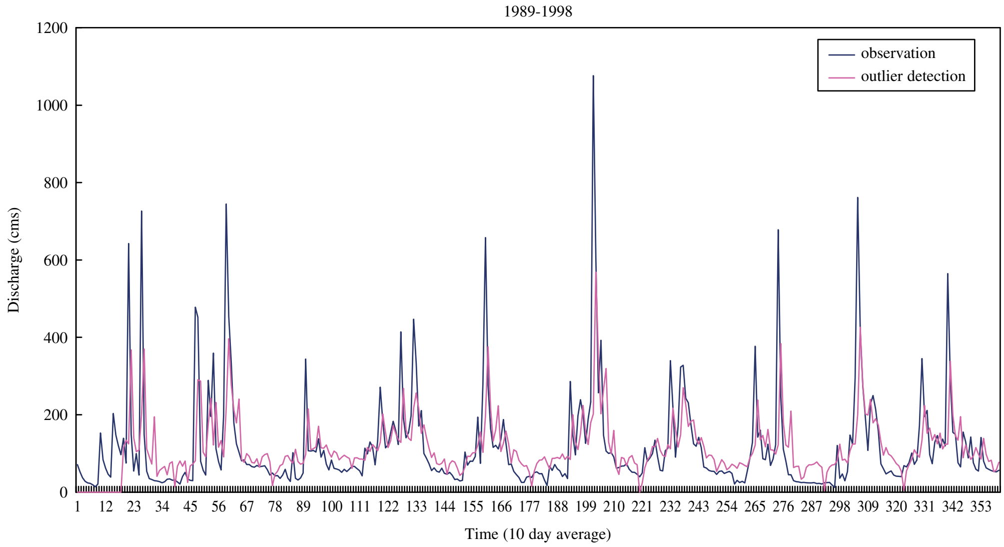

Figure 8. Calibrated results for 10-day streamflows between 1989 and 1998 using the outlier detection model

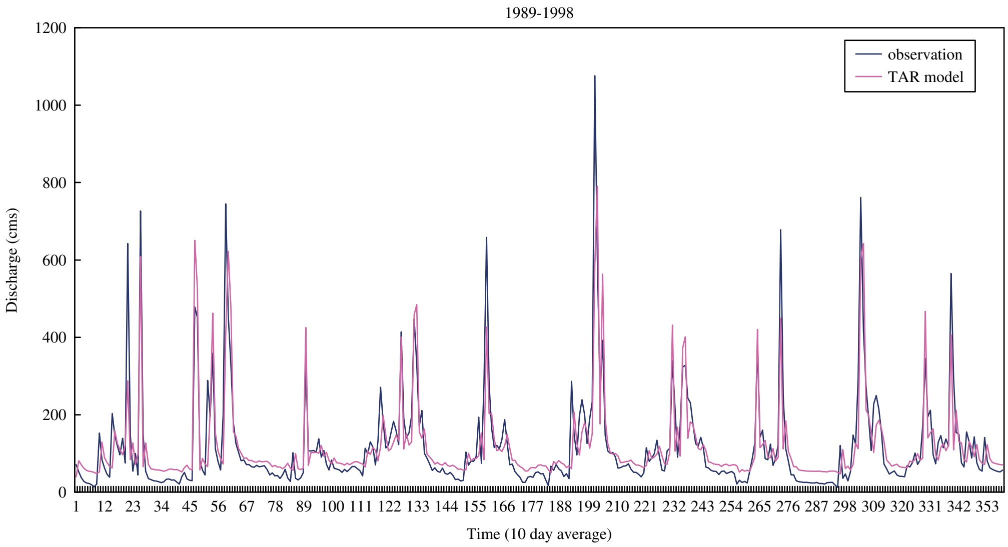

Figure 9. Calibrated results for 10-day streamflows between 1989 and 1998 using the TAR model

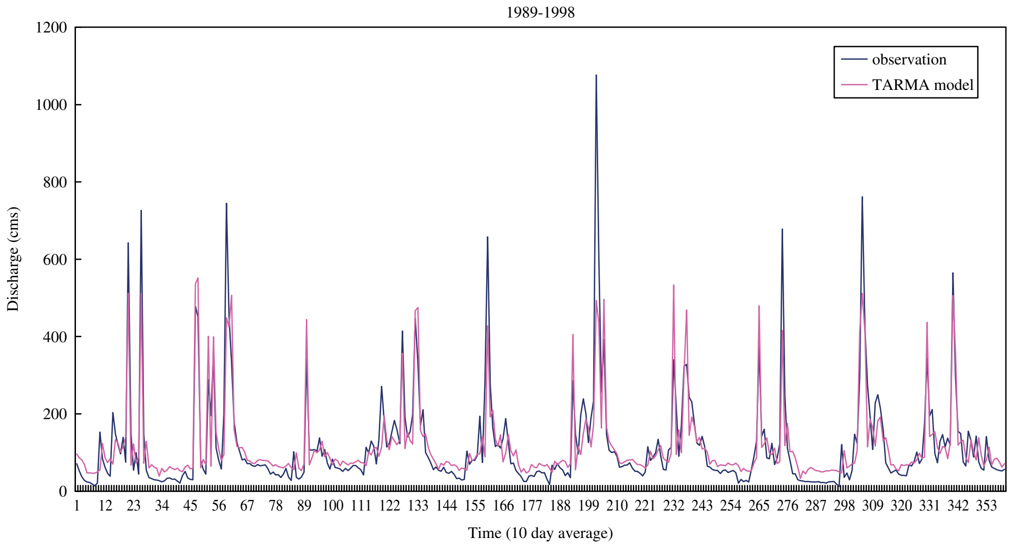

Figure 10. Calibrated results for 10-day streamflows between 1989 and 1998 using the TARMA model

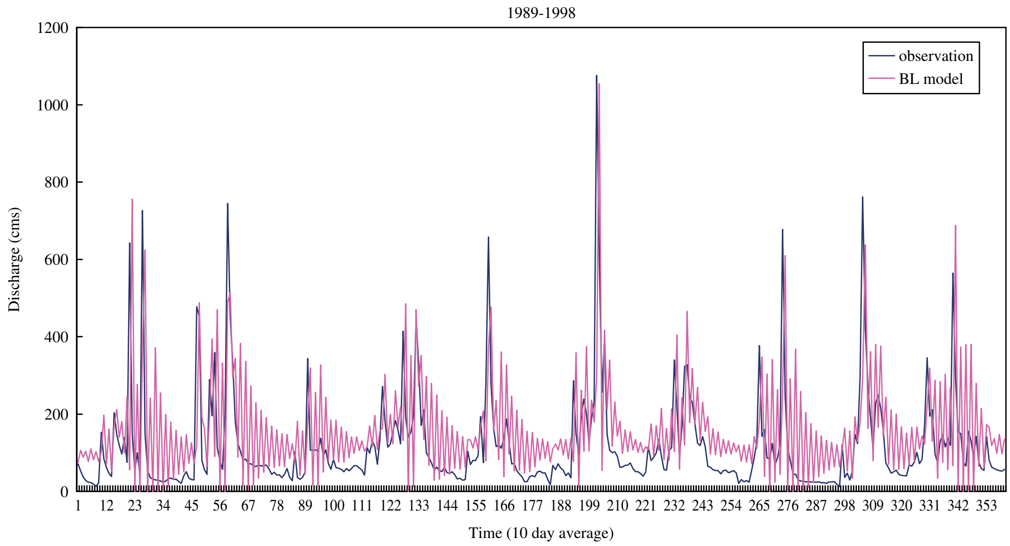

Figure 11. Calibrated results for 10-day streamflows between 1989 and 1998 using the BL model

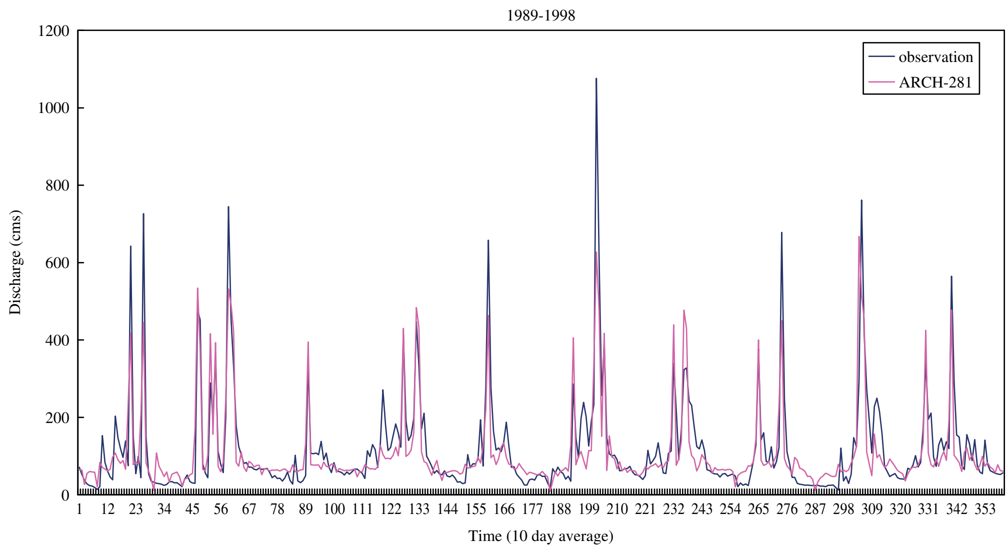

Figure 12. Calibrated results for 10-day streamflows between 1989 and 1998 using the ARCH-281 model

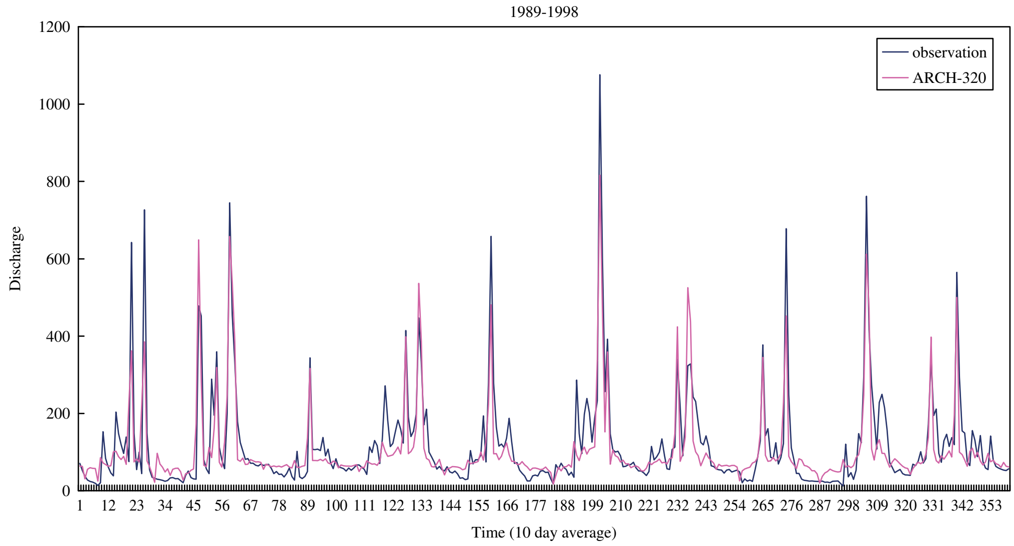

Figure 13. Calibrated results for 10-day streamflows between 1989 and 1998 using the ARCH-320 model

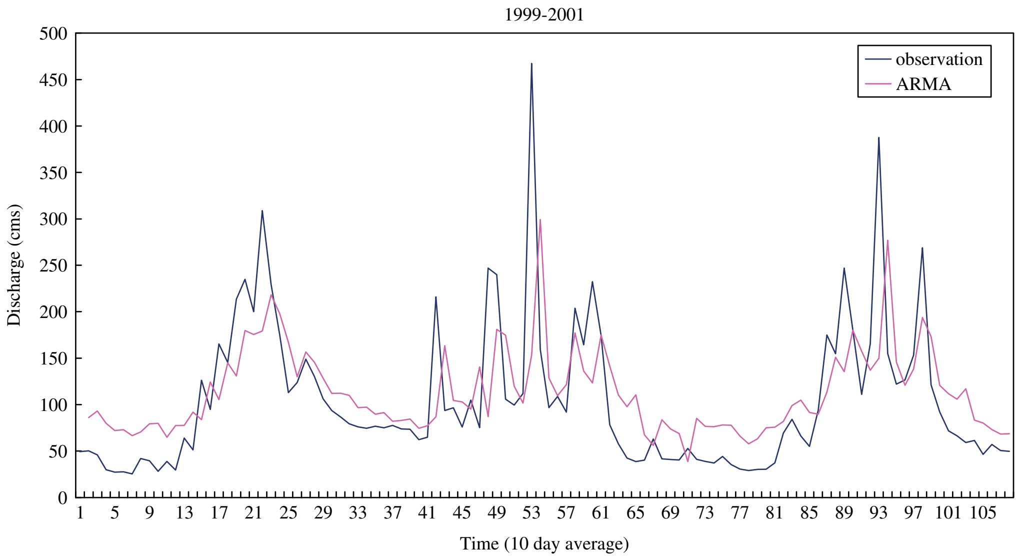

Figure 14. Verified results for 10-day streamflows between 1999 and 2001 using the ARMA model

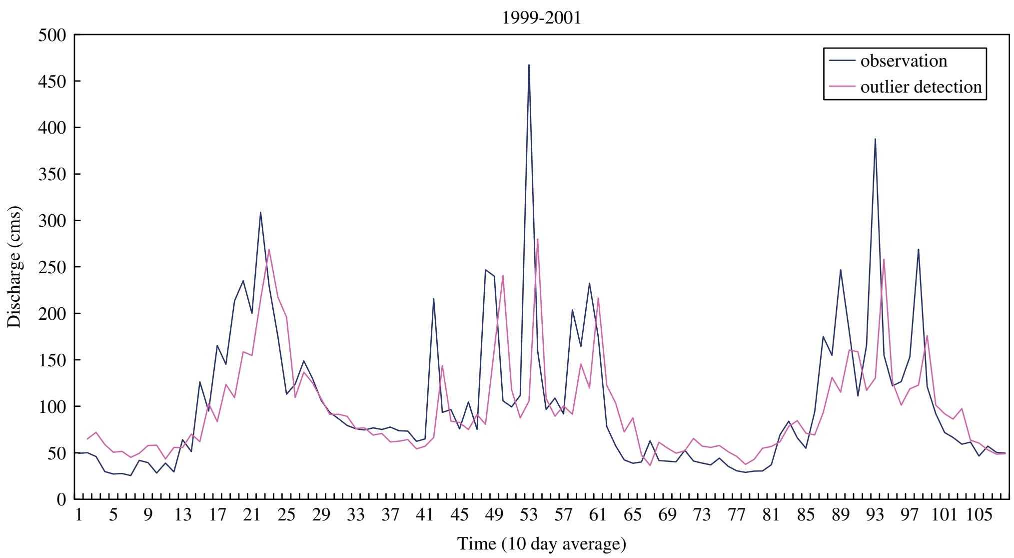

Figure 15. Verified results for 10-day streamflows between 1999 and 2001 using the outlier detection model

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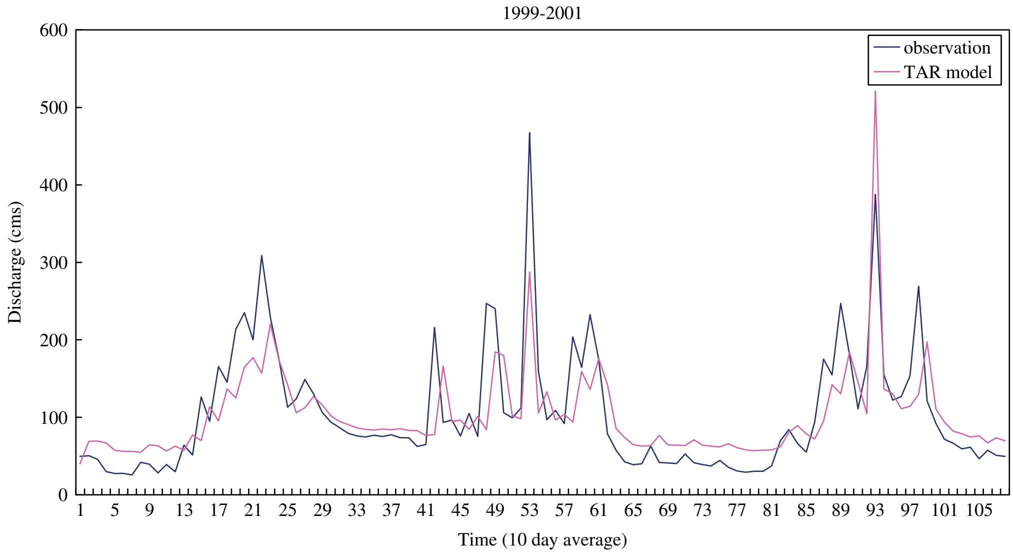

Figure 16. Verified results for 10-day streamflows between 1999 and 2001 using the TAR model

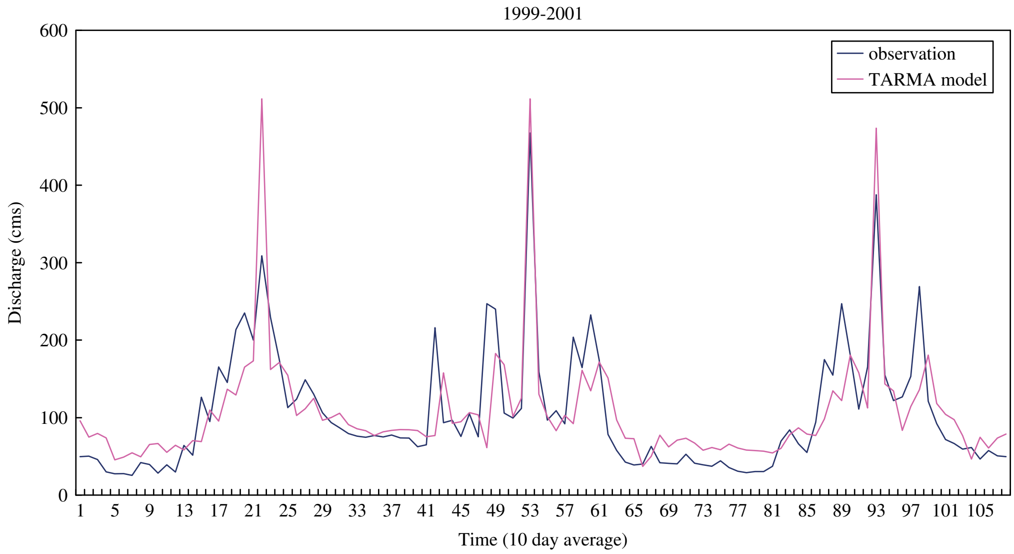

Figure 17. Verified results for 10-day streamflows between 1999 and 2001 using the TARMA model

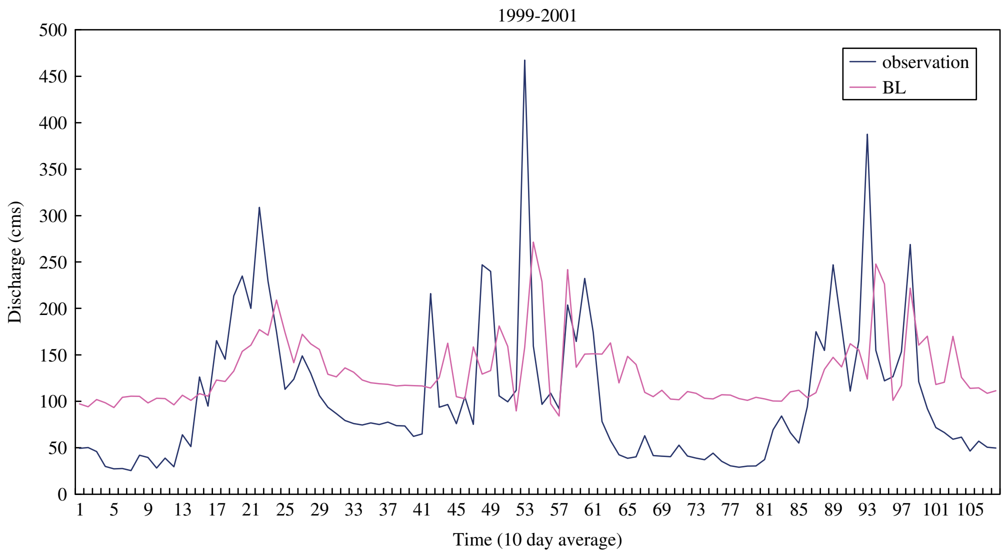

Figure 18. Verified results for 10-day streamflows between 1999 and 2001 using the BL model

DOI: 10.1002/hyp

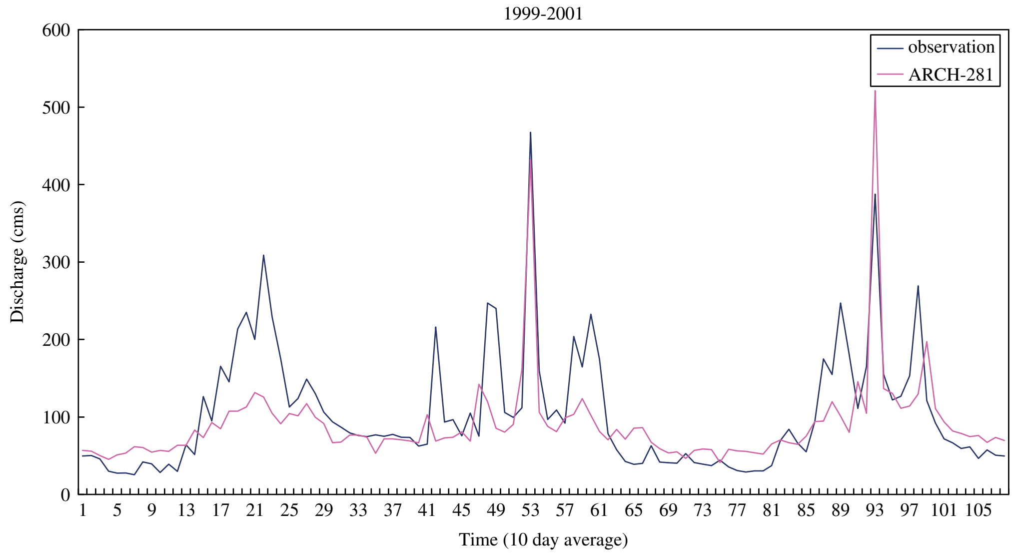

Figure 19. Verified results for 10-day streamflows between 1999 and 2001 using the ARCH-281 model

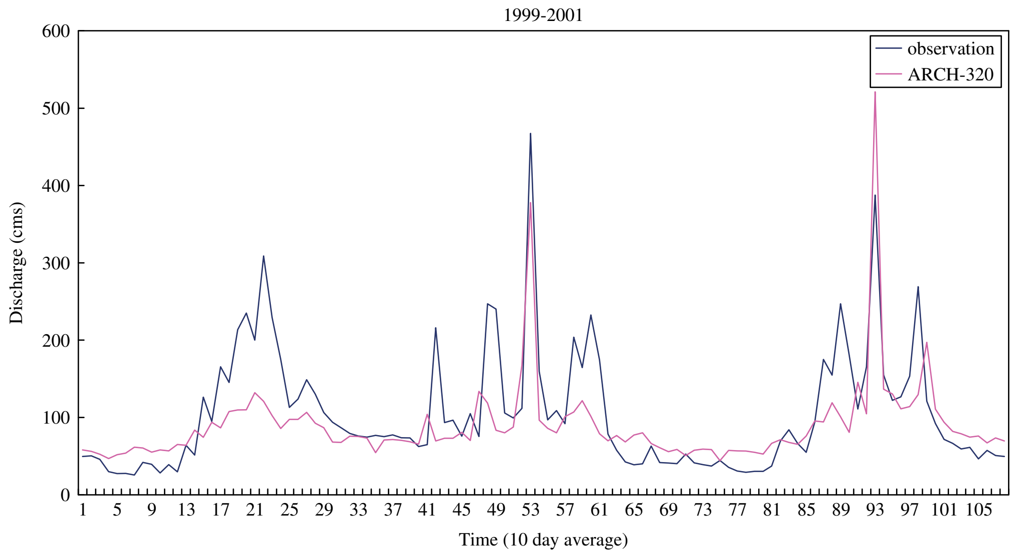

Figure 20. Verified results for 10-day streamflows between 1999 and 2001 using the ARCH-320 model

TAR, and ARCH performed better in calibration and verification. However, with regard to forecast capacity PC and EC, TAR and TARMA performed better with values of PC and EC larger than 0·5 and 0·8, in both models.

(5) Even though the multiplied oscillation effect of the BL model produces a better performance regarding peak discharge simulation, it cannot represent all data equally well because data during the dry season do not change much. Therefore, verification standards cannot be satisfied for both wet and dry seasons.

(6) The indicator *ETp* in Table 2 and Table 3 showed that the linear time series models have the characteristic of one time lag. The nonlinear time series model ARCH-320 in calibration, could solve the problem of one time lag. When the hydraulic 10-day streamflow series has multiple peaks, the TAR, TARMA, and ARCH-281 models may find the wrong peak in the wet season.

Relationships between each model and analysis system

(1) The ARMA and outlier detection models presented significant differences at the 18th order, and a negative relation as can be seen in Equations (17) and (18). Half-year periods were observed in the 10-day streamflows, representing the dry and the wet seasons and showing the seasonal change.

(2) In the TAR model, by using the CE value as a fitness function for parameter optimization, better performance was obtained for wet and dry seasons, but not for the simulation of peaks. However, peaks caused by typhoons and/or stormwater are outliers. Moreover, in this study long-term MA parameters were added to the TAR model in order to improve this model. The resulting TARMA model showed positive effects with regard to calibration, but negative effects withn regard to verification. Good simulation of whole watershed streamflow

Table IV. Comparison results of each model

	Model	Properties	Characteristics	Model program	Result	Model Application
1	ARMA	Linear model	Easy and stationary model	Easy	Not good	
2	Outlier	Linear model	Can detect outliers	Easy	Not good	
3	TAR	Nonlinear model	States the wet and dry data by using different linear models	A little complex	Good	Whole watershed streamflow forecasting
4	TARMA	Nonlinear model	States the wet and dry data by using different linear models	A little complex	Good	Whole watershed streamflow forecasting
5	ARCH-320	Nonlinear model	Utilizes residual variances and adds an explanation variable to simulate inconsistent variance data	Complex	Good	Whole watershed streamflow forecasting
6	ARCH-281	Nonlinear model	Utilizes residual variances and adds an explanation variable to simulate inconsistent variance data	Complex	Good	Whole watershed streamflow forecasting
7	BL	Nonlinear model	Cross of AR and MA can produce amplitude of oscillation	A little complex	Not good	High flow and peak value forecasting

forecasting was obtained with the TAR and TARMA models so that if one requires to separate high and low streamflows, these two models should be used.

(3) In the BL model, a combination of AR and MA resulted in oscillation of the streamflows, showing much higher peaks during the wet season. Although all streamflows were positive, the amplitudes of the BL model were so large that they resulted in negative oscillations at times, thus overfitting the performance. 10-day streamflow simulation and verification were both not good enough. If consideration of the flood case and the influence of the flood peak is of prime concern, then the BL model best captures high peak values, and could help in mitigating the consequences of floods.

(4) The TAR and TARMA models have the same problems in simulating the physical properties of maximum peaks during the wet season. Consequently, this study attempted to solve the problem of inconsistent variance by using the ARCH model. However, the ARCH model showed a higher residual variance caused by the higher peaks. This caused divergence in the simulation and the null hypothesis was rejected. Hence, this study again separated the peaks by a threshold, and then added an explanation variable to simulate the high streamflows and high variance. However, the results were still not stable even when the residual variance of the ARCH model was reduced. Thus, the results of simulation and verification did not improve much. The results of hydrograph verification were not ideal because the explanation variable caused some overforecasting of streamflows.

CONCLUSIONS

(1) A nonlinear model is better able to follow variations than a linear model.

(2) TARMA is simple to maintain and apply. Owing to the use of 10-day streamflows for water resources management, forecasting total series amounts is much more important than peak forecasting. The TARMA model showed the best performance with regard to CE, MAPE, and MAE as verification, compared with the forecast capacity PC and EC.

(3) The characteristics of 10-day streamflows in wet and dry seasons exhibited differences in the calibration and verification of the piecewise linear models TAR, TARMA, and ARCH. Figures 9, 10, 12, 13, 16, 17, 19 and 20, show better fitness for dry seasons than for wet seasons.

(4) When considering overall streamflow amounts, use of the TAR and TARMA models is recommended.

(5) When considering peak streamflow values, use of the BL model is recommended.

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